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LETTER TO THE EDITOR

Generation of an energy gap in the spectrum of two-dimensional magnetoexcitons

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Abstract
The Bethe–Salpeter equation for two-dimensional excitons in the presence of a strong external constant magnetic field is solved in the lowest-Landau-level approximation. It is shown that the constant magnetic field leads to the generation of an energy gap in the exciton spectrum.

Recently, the phenomenon of magnetic catalysis, namely the generation of an energy gap in the system of fermions in an external constant magnetic field for any arbitrary weak attractive interaction between fermions, has been considered in QED (Gusynin *et al* 1994, 1995). It is the purpose of this work to analyse the same phenomenon from the exciton point of view. We will consider two-dimensional (2D) excitons in direct-gap semiconductors with non-degenerate and isotropic bands in the presence of a constant magnetic field $\boldsymbol{B} = B\boldsymbol{z} = (0, 0, B)$ along the z-direction. The dispersion laws for the electrons and holes are $E_c(\boldsymbol{k}) = E_g + \boldsymbol{k}^2/2m_c$ and $E_v(\boldsymbol{k}) = \boldsymbol{k}^2/2m_v$, respectively. Here m_c and m_v are the corresponding effective masses and E_g is the semiconductor band gap (we set $\hbar = 1$ throughout this letter). The energy gap generation for a system of excitons can be analysed by considering the homogeneous Bethe–Salpeter (BS) equation (Bethe and Salpeter 1951, Gell-Mann and Low 1951) for the BS wavefunction $\Psi(\boldsymbol{r}_c, t_1; \boldsymbol{r}_v, t_2)$. In the case of electron–hole bound states in semiconductors with non-degenerate and isotropic bands, the BS equation has the form

$$\left(\mathrm{i}\frac{\partial}{\partial t_1} - E_g - \frac{1}{2m_c}\left[-\mathrm{i}\nabla_{\boldsymbol{r}_c} + \frac{e}{c}\boldsymbol{A}(\boldsymbol{r}_c)\right]^2\right)\left(\mathrm{i}\frac{\partial}{\partial t_2} - \frac{1}{2m_v}\left[-\mathrm{i}\nabla_{\boldsymbol{r}_v} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{r}_v)\right]^2\right) \times \Psi(\boldsymbol{r}_c, t_1; \boldsymbol{r}_v, t_2) = -\mathrm{i}I_C(\boldsymbol{r}_c, t_1; \boldsymbol{r}_v, t_2)\Psi(\boldsymbol{r}_c, t_1; \boldsymbol{r}_v, t_2) \qquad (1a)$$

where the irreducible kernel I_C represents the Coulomb attraction between electrons and holes that constitute the excitons:

$$I_C(\boldsymbol{r}_c, t_1; \boldsymbol{r}_v, t_2) = 2\pi e^2 \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{1}{q} \varepsilon^{-1}(\boldsymbol{q}, \omega) \exp\{\mathrm{i}[\boldsymbol{q}\cdot(\boldsymbol{r}_c - \boldsymbol{r}_v) - \omega(t_1 - t_2)]\}. \qquad (1b)$$

Here $\varepsilon^{-1}(\boldsymbol{q},\omega)$ is the inverse dielectric constant, $\boldsymbol{r}_{\rm c}$ and $\boldsymbol{r}_{\rm v}$ are the 2D electron and hole radius vectors and $\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{B}\times\boldsymbol{r}/2$ is the vector potential of the constant magnetic field. We note that the above-mentioned effect does not exist if we obtain the magnetoexciton spectrum $E(\boldsymbol{Q},\boldsymbol{B})$ by solving the corresponding Schrödinger equation in the lowest-Landau-level (LLL) approximation. In this approximation one can ignore transitions between Landau levels and consider only the states on the lowest Landau level. In the LLL approximation the magnetoexciton spectrum for small wavevectors $\boldsymbol{Q}$ is found to be parabolic: $E(\boldsymbol{Q},\boldsymbol{B}) = E_{\rm g}+\Omega/2-E_b+\boldsymbol{Q}^2/2M_B$ (Lozovik and Yudson 1975, 1976, Lerner and Lozovik 1980). Here $\Omega = eB/\mu c$ is the exciton cyclotron energy, defined in terms of the exciton reduced mass $\mu = m_{\rm c}m_{\rm v}/(m_{\rm c}+m_{\rm v})$, $E_b = \sqrt{\pi/2}(e^2/\varepsilon_0 R)$ is the magnetoexciton binding energy, ε_0 is the optical dielectric constant, $R = (c/eB)^{1/2}$ is the magnetic length and $M_B = 2/(E_b R^2)$, acting as the effective exciton mass.

By using the relative, $\boldsymbol{r} = \boldsymbol{r}_{\rm c} - \boldsymbol{r}_{\rm v}$, and centre-of-mass, $\boldsymbol{R} = (m_{\rm c}\boldsymbol{r}_{\rm c} + m_{\rm v}\boldsymbol{r}_{\rm v})/(m_{\rm c}+m_{\rm v})$, coordinates, the exciton wavefunction $\Psi(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)$ can be written in the form

$$\Psi(\boldsymbol{r}_{\rm c},t_1;\boldsymbol{r}_{\rm v},t_2) = \exp\left\{{\rm i}\left[\boldsymbol{Q}\cdot\boldsymbol{R} - \frac{e}{c}\boldsymbol{r}\cdot\boldsymbol{A}(\boldsymbol{R}) - E(\alpha_{\rm c}t_1+\alpha_{\rm v}t_2)\right]\right\}\Phi_{\boldsymbol{Q}}(\boldsymbol{r},t_1-t_2) \tag{2}$$

where $E \equiv E(\boldsymbol{Q},\boldsymbol{B})$, $\alpha_{c,{\rm v}} = m_{c,{\rm v}}/M$ and $M = m_{\rm c}+m_{\rm v}$ is the exciton mass. It is more convenient to write the BS equation (1*a*) in the momentum space, taking into account the one-particle band structure of the semiconductor. For this reason we introduce the Fourier transform of the function $\Phi_{\boldsymbol{Q}}(\boldsymbol{r},t)$ of the relative motion:

$$\Phi_{\boldsymbol{Q}}(\boldsymbol{r},t) = \int\frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\int_{-\infty}^{+\infty}\frac{{\rm d}\omega}{2\pi}\exp\{{\rm i}[\boldsymbol{q}\cdot\boldsymbol{r}-\omega t]\}\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega). \tag{3}$$

After some tedious but straightforward calculations, one can obtain the following BS equation for the function $\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega)$:

$$\begin{aligned}&[\Omega-\Omega_{\rm c}(\boldsymbol{p},\boldsymbol{Q})][\Omega-\Omega_{\rm v}(\boldsymbol{p},\boldsymbol{Q})]\Phi_{\boldsymbol{Q}}(\boldsymbol{p},\Omega)\\ &\qquad = -{\rm i}\int\frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\int_{-\infty}^{+\infty}\frac{{\rm d}\omega}{2\pi}I(\boldsymbol{p},\boldsymbol{q},\boldsymbol{Q};\Omega,\omega)\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega)\end{aligned} \tag{4a}$$

where the kernel of the BS equation can be written as

$$I(\boldsymbol{p},\boldsymbol{q},\boldsymbol{Q};\Omega,\omega) = I_{\rm C}(|\boldsymbol{q}-\boldsymbol{p}|;\Omega-\omega) + I_B(\boldsymbol{p},\boldsymbol{q},\boldsymbol{Q};\Omega,\omega). \tag{4b}$$

The Coulomb part of the kernel $I_{\rm C}$ is given by the well-known expression

$$I_{\rm C}(\boldsymbol{p};\omega) = \frac{2\pi e^2}{|\boldsymbol{p}|}\varepsilon^{-1}(\boldsymbol{p},\omega). \tag{4c}$$

The magnetic part of the kernel I_B is due to the interaction of the electron and hole with the magnetic field:

$$\begin{aligned}I_B(\boldsymbol{p},\boldsymbol{q},\boldsymbol{Q};\Omega,\omega) &= -2\pi{\rm i}\delta(\omega-\Omega)\{[\Omega-\Omega_{\rm c}(\boldsymbol{q},\boldsymbol{Q})]\Omega_{\rm v}^B(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q})\\ &\quad -[\Omega-\Omega_{\rm v}(\boldsymbol{q},\boldsymbol{Q})]\Omega_{\rm c}^B(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q}) + \Omega_{\rm cv}^B(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q})\}.\end{aligned} \tag{4d}$$

Here the following notation has been used:

$$\Omega_{\rm c}(\boldsymbol{q},\boldsymbol{Q}) = E_{\rm c}(\boldsymbol{q}+\alpha_{\rm c}\boldsymbol{Q}) - \alpha_{\rm c}E \qquad \Omega_{\rm v}(\boldsymbol{q},\boldsymbol{Q}) = -E_{\rm v}(\boldsymbol{q}-\alpha_{\rm v}\boldsymbol{Q}) + \alpha_{\rm v}E \tag{5a}$$

$$\Omega_{\rm c}^B(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q}) = \int{\rm d}^2\boldsymbol{r}\,{\rm e}^{{\rm i}(\boldsymbol{q}-\boldsymbol{p})\cdot\boldsymbol{r}}\left\{\frac{e}{2Mc}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{Q} + \frac{e}{2m_{\rm c}c}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{q} + \frac{e^2}{8m_{\rm c}c^2}(\boldsymbol{B}\times\boldsymbol{r})^2\right\} \tag{5b}$$

$$\Omega_{\rm v}^{B}(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q}) = \int {\rm d}^2\boldsymbol{r}\,{\rm e}^{{\rm i}(\boldsymbol{q}-\boldsymbol{p})\cdot\boldsymbol{r}}\left\{\frac{e}{2Mc}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{Q} - \frac{e}{2m_{\rm v}c}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{q} + \frac{e^2}{8m_{\rm v}c^2}(\boldsymbol{B}\times\boldsymbol{r})^2\right\} \tag{5c}$$

$$\begin{aligned}\Omega_{\rm cv}^{B}(\boldsymbol{q},\boldsymbol{p},\boldsymbol{Q}) = \int {\rm d}^2\boldsymbol{r}\,{\rm e}^{{\rm i}(\boldsymbol{q}-\boldsymbol{p})\cdot\boldsymbol{r}}\Bigg\{\Bigg[&\frac{e}{2Mc}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{Q}\\ &- \frac{e}{2m_{\rm v}c}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{q} + \frac{e^2}{8m_{\rm v}c^2}(\boldsymbol{B}\times\boldsymbol{r})^2\Bigg]\Bigg[\frac{e}{2Mc}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{Q}\\ &+ \frac{e}{2m_{\rm c}c}(\boldsymbol{B}\times\boldsymbol{r})\cdot\boldsymbol{q} + \frac{e^2}{8m_{\rm c}c^2}(\boldsymbol{B}\times\boldsymbol{r})^2\Bigg]\Bigg\}.\end{aligned} \tag{5d}$$

One of the main peculiarities which distinguishes the BS equation from the Schrödinger equation is the dependence of the wavefunction (2) on the relative time. In the absence of a magnetic field the physical source for its appearance is the retardation of the Coulomb interaction due to the frequency dependence of the inverse dielectric constant. In what follows we will use the instantaneous approximation for the Coulomb interaction assuming that $\varepsilon^{-1}(\boldsymbol{q},\omega) \approx \varepsilon_0^{-1}$. The magnetic field is the second source for the essential relative time dependence of the BS wavefunction. The physical reason for this can be intuitively understood as the result of the Lorentz force tending to separate the charges in each exciton.

As in the Wick–Cutkovsky model (Wick 1954), we are looking for a solution of the BS equation (4a) of the form

$$\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega) = {\rm i}f_{\boldsymbol{Q}}(\boldsymbol{q},\omega)\left[\frac{1}{\omega - \Omega_{\rm c}(\boldsymbol{q},\boldsymbol{Q}) + {\rm i}\delta} - \frac{1}{\omega - \Omega_{\rm v}(\boldsymbol{q},\boldsymbol{Q}) - {\rm i}\delta}\right] \tag{6a}$$

where $f_{\boldsymbol{Q}}(\boldsymbol{q},\omega)$ is a regular function in the frequency plane with the following property: $f_{\boldsymbol{Q}}(\boldsymbol{q},\Omega_{\rm v}(\boldsymbol{Q},\boldsymbol{q})) = f_{\boldsymbol{Q}}(\boldsymbol{q},\Omega_{\rm c}(\boldsymbol{Q},\boldsymbol{q}))$. Let us define the function $\Phi_{\boldsymbol{Q}}(\boldsymbol{q})$, which is the Fourier transform of the exciton wavefunction of the relative motion $\Phi_{\boldsymbol{Q}}(\boldsymbol{r}) = \Phi_{\boldsymbol{Q}}(\boldsymbol{r}, t_1 - t_2 = 0)$:

$$\Phi_{\boldsymbol{Q}}(\boldsymbol{r}) = \int_{-\infty}^{+\infty}\frac{{\rm d}\omega}{2\pi}\,\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega). \tag{6b}$$

Thus, by taking into account the analytic properties of $\Phi_{\boldsymbol{Q}}(\boldsymbol{q},\omega)$, one can obtain the following BS equation for determining the exciton energy $E = E(\boldsymbol{Q},\boldsymbol{B})$ and the Fourier transform $\Phi_{\boldsymbol{Q}}(\boldsymbol{q})$ of the exciton wavefunction:

$$\begin{aligned}&\left[E - E_{\rm g} - \frac{\boldsymbol{Q}^2}{2M} - \frac{\boldsymbol{p}^2}{2\mu}\right]\Phi_{\boldsymbol{Q}}(\boldsymbol{p}) + \frac{2\pi e^2}{\varepsilon_0}\int\frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\,\frac{1}{|\boldsymbol{p}-\boldsymbol{q}|}\Phi_{\boldsymbol{Q}}(\boldsymbol{q}) + \int\frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\,\Phi_{\boldsymbol{Q}}(\boldsymbol{q})\\ &\qquad\times\left\{\frac{[E - E_{\rm c}(\boldsymbol{q}+\alpha_{\rm c}\boldsymbol{Q}) - E_{\rm v}(\boldsymbol{q}-\alpha_{\rm v}\boldsymbol{Q})]\Omega_{\rm c}^{B}(\boldsymbol{Q},\boldsymbol{q},\boldsymbol{p}) + \Omega_{\rm cv}^{B}(\boldsymbol{Q},\boldsymbol{q},\boldsymbol{p})}{E - E_{\rm c}(\boldsymbol{q}+\alpha_{\rm c}\boldsymbol{Q}) - E_{\rm v}(\boldsymbol{p}-\alpha_{\rm v}\boldsymbol{Q}) + {\rm i}\delta}\right.\\ &\qquad\times\left.\frac{[E - E_{\rm c}(\boldsymbol{q}+\alpha_{\rm c}\boldsymbol{Q}) - E_{\rm v}(\boldsymbol{q}-\alpha_{\rm v}\boldsymbol{Q})]\Omega_{\rm v}^{B}(\boldsymbol{Q},\boldsymbol{q},\boldsymbol{p}) + \Omega_{\rm cv}^{B}(\boldsymbol{Q},\boldsymbol{q},\boldsymbol{p})}{E - E_{\rm c}(\boldsymbol{p}+\alpha_{\rm c}\boldsymbol{Q}) - E_{\rm v}(\boldsymbol{q}-\alpha_{\rm v}\boldsymbol{Q}) + {\rm i}\delta}\right\} = 0.\end{aligned} \tag{7}$$

In what follows we will use the magnetic length R for the unit length and the exciton cyclotron energy Ω for the energy unit. In the coordinate space the exciton wavefunction has the form

$$\Phi_{\boldsymbol{Q}}(\boldsymbol{r}) = \int\frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\exp[{\rm i}(\boldsymbol{q}\cdot\boldsymbol{r})]\Phi_{\boldsymbol{Q}}(\boldsymbol{q}) = \exp\left[{\rm i}\frac{\gamma}{2}\boldsymbol{Q}\cdot\boldsymbol{r}\right]\Psi(\boldsymbol{r}-\boldsymbol{R}_0). \tag{8a}$$

Here $\boldsymbol{R}_0 = \boldsymbol{z} \times \boldsymbol{Q}$ and the function Ψ satisfies the following BS equation:

$$\left[-\frac{1}{2}\nabla^2_{r_\perp} - \left(\frac{R}{a_0}\right)\frac{1}{|\boldsymbol{r}+\boldsymbol{R}_0|} - \frac{\mathrm{i}\gamma}{2}(\boldsymbol{z}\times\boldsymbol{r})\cdot\nabla_r + \frac{1}{8}\boldsymbol{r}^2\right]\Psi(\boldsymbol{r})$$
$$+\,\mathrm{e}^{-\mathrm{i}(\gamma/2)\boldsymbol{r}\cdot\boldsymbol{Q}}\int \mathrm{d}^2\boldsymbol{r}_1\, V_{\mathrm{eff}}(\boldsymbol{r}+\boldsymbol{R}_0;\boldsymbol{r}_1+\boldsymbol{R}_0;\boldsymbol{Q};\varepsilon_B)\mathrm{e}^{\mathrm{i}(\gamma/2)\boldsymbol{r}_1\cdot\boldsymbol{Q}}\Psi(\boldsymbol{r}_1) = \varepsilon_B\Psi(\boldsymbol{r}) \tag{8b}$$

where

$$V_{\mathrm{eff}}(\boldsymbol{r};\boldsymbol{r}_1;\boldsymbol{Q};E) = \int \frac{\mathrm{d}^2\boldsymbol{p}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\mathrm{d}^2\boldsymbol{R}\exp\{\mathrm{i}[(\boldsymbol{r}-\boldsymbol{R})\cdot\boldsymbol{p}-(\boldsymbol{r}_1-\boldsymbol{R})\cdot\boldsymbol{q}]\}$$
$$\times V_{\mathrm{eff}}(\boldsymbol{p};\boldsymbol{q};\boldsymbol{Q};\boldsymbol{R};E) \tag{9a}$$

$$V_{\mathrm{eff}}(\boldsymbol{p};\boldsymbol{q};\boldsymbol{Q};\boldsymbol{R};E) = \frac{\alpha_{\mathrm{c}}\alpha_{\mathrm{v}}}{4}\Bigg\{\{[\boldsymbol{p}^2-\boldsymbol{q}^2]+2\alpha_{\mathrm{c}}(\boldsymbol{p}-\boldsymbol{q})\cdot\boldsymbol{Q}+\boldsymbol{z}\cdot(\boldsymbol{R}\times(\boldsymbol{q}+\alpha_{\mathrm{c}}\boldsymbol{Q}))+\tfrac{1}{4}\boldsymbol{R}^2\}$$
$$\times\{-\boldsymbol{z}\cdot(\boldsymbol{R}\times(\boldsymbol{q}-\alpha_{\mathrm{v}}\boldsymbol{Q}))+\tfrac{1}{4}\boldsymbol{R}^2\}$$
$$\times\left[E-\frac{\alpha_{\mathrm{v}}}{2}(\boldsymbol{p}+\alpha_{\mathrm{c}}\boldsymbol{Q})^2-\frac{\alpha_{\mathrm{c}}}{2}(\boldsymbol{q}-\alpha_{\mathrm{v}}\boldsymbol{Q})^2+\mathrm{i}\delta\right]^{-1}$$
$$+\{[\boldsymbol{p}^2-\boldsymbol{q}^2]+2\alpha_{\mathrm{v}}(\boldsymbol{p}-\boldsymbol{q})\cdot\boldsymbol{Q}-\boldsymbol{z}\cdot(\boldsymbol{R}\times(\boldsymbol{q}-\alpha_{\mathrm{v}}\boldsymbol{Q}))+\tfrac{1}{4}\boldsymbol{R}^2\}$$
$$\times\{\boldsymbol{z}\cdot(\boldsymbol{R}\times(\boldsymbol{q}+\alpha_{\mathrm{c}}\boldsymbol{Q}))+\tfrac{1}{4}\boldsymbol{R}^2\}$$
$$\times\left[E-\frac{\alpha_{\mathrm{v}}}{2}(\boldsymbol{q}+\alpha_{\mathrm{c}}\boldsymbol{Q})^2-\frac{\alpha_{\mathrm{c}}}{2}(\boldsymbol{p}-\alpha_{\mathrm{v}}\boldsymbol{Q})^2+\mathrm{i}\delta\right]^{-1}\Bigg\}. \tag{9b}$$

Here $\gamma = (m_{\mathrm{v}} - m_{\mathrm{c}})/M$, $\varepsilon_B = (E - E_{\mathrm{g}})/\Omega$ and $a_0 = \varepsilon_0/(\mu e^2)$ is the exciton Bohr radius. Equation (8*b*) is the BS equation for the exciton wavefunction. In comparison to the Schrödinger equation, the above BS equation contains additionally an effective interaction V_{eff}, which depends on the exciton energy $E(\boldsymbol{Q},\boldsymbol{B})$. As can be seen, the main contribution in the integrals on the right-hand side of equation (9*b*) is formed in the region $\boldsymbol{q}\approx\boldsymbol{p}$, so we can use the following expression for the effective interaction V_{eff}:

$$V_{\mathrm{eff}}(\boldsymbol{r};\boldsymbol{r}_1;\boldsymbol{Q};E)\approx\int\frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\exp\{\mathrm{i}(\boldsymbol{r}-\boldsymbol{r}_1)\cdot\boldsymbol{q}\}F(\boldsymbol{q};\boldsymbol{r};\boldsymbol{Q};E) \tag{10a}$$

where

$$F(\boldsymbol{q};\boldsymbol{r};\boldsymbol{Q};E) = \frac{1}{8}(1-\gamma^2)\frac{\left[\boldsymbol{r}^2/4+\frac{1}{2}\boldsymbol{z}\cdot(\boldsymbol{r}\times\boldsymbol{Q})\right]^2-\{\boldsymbol{z}\cdot[\boldsymbol{r}\times(\boldsymbol{q}-\frac{1}{2}\gamma\boldsymbol{Q})]\}^2}{E-\frac{1}{8}(1-\gamma^2)\boldsymbol{Q}^2-\frac{1}{2}\boldsymbol{q}^2+\mathrm{i}\delta}. \tag{10b}$$

To obtain the solution of the BS equation (8*b*), we: (1) restrict our calculations to the strong-magnetic-field regime, where the LLL approximation takes place; (2) use the fact that in sufficiently strong magnetic fields the Coulomb interaction can be treated by perturbation theory. To zero order in the Coulomb interaction, one can use the function $\Psi_0(\boldsymbol{r}) = (2\pi)^{-1/2}\exp(-\boldsymbol{r}^2/4)$ as a solution of the BS equation. Thus, for small wavevectors $|\boldsymbol{Q}|R \ll 1$, we obtain the following equation for $\varepsilon_B = (E(\boldsymbol{Q},\boldsymbol{B}) - E_{\mathrm{g}})/\Omega$ in zero order in the Coulomb interaction:

$$\varepsilon_B = \varepsilon_B^{(0)}(\boldsymbol{Q}) = 1/2 + (1/8)(1-\gamma^2)\left[W_0(\varepsilon_B^{(0)}) + W_2(\varepsilon_B^{(0)})\boldsymbol{Q}^2\right]$$

where $W_0(x)$ and $W_2(x)$ are defined as follows:

$$W_0(x) = 11 - 4x - 8(1-6x+2x^2)\Gamma(-4x)\exp(-4x)$$
$$W_2(x) = 11/2 - x^{-1} - 2E - 8(1-3x+x^2)\Gamma(-4x)\exp(-4x)$$

and $\Gamma(x)$ denotes the gamma function. The numerical solution of the above equation for small vectors $\boldsymbol{Q}$ leads to the quadratic magnetoexciton spectrum $\varepsilon_B^{(0)}(\boldsymbol{Q}) \approx 1/2 + \Delta + A\boldsymbol{Q}^2$, where Δ is the energy gap generated by the magnetic field and A is the numerical coefficient. For electron–heavy-hole excitons in GaAs ($m_c = 0.067m_0$ and the Luttinger parameters $\gamma_1 = 6.9$ and $\gamma_2 = 2.4$), the calculated values are $\Delta = 0.0794$ and $A = 0.018$. The magnetoexciton spectrum for small vectors $\boldsymbol{Q}$ to first order in the Coulomb interaction is

$$E(\boldsymbol{Q}, \boldsymbol{B}) = E_g + (1/2 + \Delta)\Omega - E_b + \boldsymbol{Q}^2/(2M_{exc})$$

where the effective magnetoexciton mass is $M_{exc} = M_B M_0/(M_B + M_0)$, and $M_0 = \mu/(2A)$. For the experiments in homogeneous magnetic fields where the high-field regime is reached, $M_0 \gg M_B$ (for GaAs, $M_0 = 10M_B$ corresponds to $B \approx 20$ T) and the magnetoexciton mass is equal to M_B.

We have assumed a strict 2D case, but it is easy to generalize the calculations to the case of quantum well excitons. Although not shown, we have performed calculations (Koinov 2001) of the ground state of the exciton in a 4 nm GaAs quantum well sandwiched between two $Ga_{0.7}Al_{0.3}As$ layers. For electron–heavy-hole excitons ($\gamma_1 = 6.9$ and $\gamma_2 = 2.4$) the calculated ground-state energies in the fields of 20, 18 and 16 T are 1.645, 1.643 and 1.641 eV, respectively. There is a very good agreement between the calculations and the results of a photoluminescence study of heavy-hole excitons confined in very thin quantum wells (the observed peaks are at 1.644, 1.643 and 1.642 eV) reported by Ko *et al* (1998).

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